Noise-Induced Inflation

Esteban Calzetta[1]

We consider a closed Friedmann–Robertson–Walker universe driven by the backreaction from a massless, non-conformally coupled quantum scalar field. We show that the backreaction of the quantum field is able to drive the cosmological scale factor over the barrier of the classical potential so that if the universe starts near zero scale factor (initial singularity) it can make the transition to an exponentially expanding de Sitter phase with a probability comparable to that from quantum tunneling processes. The emphasis throughout is on the stochastic nature of backreaction, which comes from the quantum fluctuations of the fundamental fields.

1. INTRODUCTION

In this paper, based on work done in collaboration with Enric Verdaguer [1], we shall discuss a model of the early universe where a spatially closed Friedmann–Robertson–Walker universe avoids recollapse and launchs into inflationary expansion due to the effects of the backreaction of quantized matter fields. Central to the argument is the fact that this backreaction is both memory dependent and to some extent random, due to quantum fluctuations of the fundamental fields. The stress on particle creation and noise is the main novelty of our approach with respect to the by now large corpus of semiclassical cosmology [2].

To the best of our knowledge, this work is the most elaborate application to date of the basic idea that quantum fluctuations of fundamental fields act on the geometry of the universe as a stochastic energy-momentum tensor, put forward by several researchers [3–9]. We shall pass briefly over the technical details, which are contained in the original work, to concentrate on the physical ideas. In next section, we give a general discussion of why and

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[1] Instituto de Astronomía y Física del Espacio (IAFE) and Departamento de Física, Universidad de Buenos Aires, Buenos Aires, Argentina.

how stochastic terms ought to be included in Einstein equations; the subsequent section describes the model and the development of its solution, and we conclude with some brief final remarks.

2. THE SEMICLASSICAL APPROXIMATION: PARTICLE CREATION AND NOISE

Ever since the development of quantum mechanics and relativity theory early in this century, their final unification has been one of the most sought after prizes of theoretical physics. Moreover, as Hubble's observations and then the discovery of the cosmic microwave radiation has taught us that the structure of our universe is determined by events at the very beginning of its evolution, this search for unification acquired more than academic interest, since these early stages presumably demand a quantum description. However, in spite of some progress, especially in the 1960s, the goal of a quantum theory of gravity seems now as elusive as ever (see refs. 10 for early approaches).

In the meantime, it has been realized that even without a full theory it was possible to find answers to most pressing questions, at least from the cosmological and astrophysical viewpoints. Basically the theoretical expectation is that quantum gravitational effects will become relevant at the Planck scale (of 10^{19} GeV in natural units), while quantum effects associated with matter are conspicuous at the scales associated with the masses of those particles, orders of magnitude below that. Therefore, it makes sense to develop models where the geometry of the universe (or, say, the space surrounding a compact object) is treated in the terms of Einstein's theory (which is to say, classically) while matter is quantized [2]. As shown by Parker [11], even these restricted models leave ample room for exotic behavior, probably its most spectacular manifestation being Hawking radiation [12, 13].

Now, there being no question that quantum fluctuations *weigh*, as shown by the Casimir effect, their presence should affect the evolution of the said geometry, on equal footing with other forms of matter, gravity included. So, although models of quantum test fields on given backgrounds have been immensely useful to clarify the formal aspects of the theory, the real goal is to develop self-consistent models where the feedback between the quantum matter and the classical geometry is fully accounted for. This raises the issue of how the quantum matter affects the geometry.

The simplest and earliest answer to this question is that it is possible that while the field amplitudes associated with matter are to be described by operators in some functional space, other observables such as energy-momentum, which are composites of field operators, may develop a condensate or *c*-number part. In the simplest case, this condensate may be computed as the expectation value of the corresponding composite operator in an ade-

quate quantum state, and this expectation value is to be included as the quantum matter contribution to Einstein's equations [14].

By now, there is a substantial body of work showing that this approach can be made mathematically consistent [2]. However, there are also serious doubts concerning whether it is *physically* satisfactory [15, 16]. One can formulate these doubts with various degrees of sophistication, but the basic argument goes as follows.

In a typical self-consistent evolution problem, the identification of the energy-momentum tensor of the matter fields involves a normal ordering or subtraction procedure, to take care of the ever-present divergences of quantum field theory. Suppose this normal ordering amounts to subtracting the energy-momentum defined with respect to some local vacuum state, the most sensible choice being a local adiabatic vacuum of some order. Suppose we let the model evolve from some time t_i to some final time t_f. In almost every interesting case, the evolving geometry will mix the positive- and negative-frequency components of the field operator, so that the creation and destruction operators of the adiabatic model at time t_f will be related to that at time t_i by a Bogolubov transformation. This transformation is characterized by two complex parameters, α and β, with $|\alpha|^2 - |\beta|^2 = 1$. If at time t_i a given one-particle state was occupied by n adiabatic particles, then at time t_f we find, on average, $n' = |\beta|^2 + n|\alpha|^2$ particles there (we assume Bose statistics, for concreteness). This average is reflected in the evolution of the mean energy-momentum. But the state at time t_f will not, in general, be a state with a well-defined occupation number (certainly not, if the state at time t_i was), and when we look at the dispersion in particle number, we see that unless $|\beta|^2 \ll n$, in which case we did not need to bother with self-consistency, the average fluctuation in particle number is of the order of the mean occupation number itself. So the average energy momentum of the created particles is only vaguely related to what might have happened "on the spot."

The issue then arises of how to introduce these fluctuations in an actual model. Maybe the final answer is that nothing short of full quantum gravity is truly satisfactory, but if one wishes to retain the classical character of the metric, then it seems that the only possibility is to add to the mean energy-momentum of the fields a stochastic component which would represent the leading effects of the quantum fluctuations.

Halliwell and others [17, 18] have developed a picture which shows that this classical, stochastic energy-momentum at least makes sense. In this approach, the classical geometry is seen as an apparatus which measures the energy-momentum of the quantum matter fields and reacts to the measured value. It is then seen that the results are *c*-numbers, but their progression in time does not follow deterministic laws. It is natural to extract the deterministic, mean evolution and call the remainder random noise. In the limit where

the measurements are inaccurate enough and repeated often enough, we may assume that the evolution of the fluctuations will not be greatly perturbed by the measurement process, and so they may be computed from ordinary quantum field-theoretic rules.

2.1. The Closed-Time-Path Effective Action

In this limit, therefore, a new paradigm appears, derived from the work of Feynman and Vernon [19]. We now regard the geometry as an *open system* evolving in the environment provided by the matter quantum fluctuations [20]. Since the detailed evolution of the latter is deemed irrelevant, our only concern is to estimate the *influence action*, namely, the modification to the gravitational action due to the influence of the environment. In the limit where the geometry is actually taken to be classical, the task becomes identical to that of computing the so-called Schwinger–Keldysh effective action [21]. We shall disregard the somewhat technical distinctions between these two objects, regarding them as the same; in a nutshell, the Schwinger–Keldish or closed-time-path (CTP) effective action (EA) is the influence action evaluated over an infinite time range, which actually takes care of some difficulties in the evaluation of the influence action over finite lapses.

The CTPEA is a truly remarkable object, which achieves the miracle of providing a well-defined variational method to derive causal but nonlocal-in-time equations of motion. It is not hard to see where the difficulty lies. Suppose you have a system described by some variable $\phi(t)$, and write for it an action functional $S[\phi]$ whose variation yields the equations of motion $\delta S/\delta\phi(t) = 0$. If the equations are causal, then $\delta^2 S/\delta\phi(t)\,\delta\phi(t') = 0$ whenever $t' > t$. Since second derivatives commute, the second derivative actually vanishes if only $t' \neq t$, and the action must be necessarily local in time. Since all dissipative effects are physically limiting cases of nonlocal time interactions (when the response time of the bath is much shorter than the characteristic time of the system), it follows more generally that there is no variational principle for dissipative, causal evolutions.

The CTPEA achieves the impossible by adding to every degree of freedom $\phi^+(t)$ a mirror degree of freedom $\phi^-(t)$, so that the CTPEA $\Gamma = \Gamma[\phi^+, \phi^-]$ and the equations of motion are $\delta\Gamma/\delta\phi^+ = 0$. The right count of degrees of freedom is restored by imposing, *after the variation has been taken*, the constraint $\phi^+ = \phi^- = \phi$, the physical degree of freedom. Causality only demands

$$\frac{\delta^2\Gamma}{\delta\phi^+(t)\,\delta\phi^+(t')} + \frac{\delta^2\Gamma}{\delta\phi^+(t)\,\delta\phi^-(t')} = 0 \qquad \text{when} \quad t < t' \qquad (1)$$

For example, suppose that the solution to the equations of motion is

just $\phi = 0$, and we seek the dynamics of small fluctuations. The only quadratic action compatible with the causality constraint has the form

$$\Gamma = \frac{1}{2} \int dt \, dt' \, \{[\phi](t)\mathcal{D}(t, t')\{\phi\}(t') + [\phi](t)\mathcal{N}(t, t')[\phi](t')\} \tag{2}$$

where $[\phi]$ eq $\phi^+ - \phi^-$; $\{\phi\} = \phi^+ + \phi^-$; and $\mathcal{D}(t, t') = 0$ if $t < t'$. The equations of motion are

$$\int dt' \, \mathcal{D}(t, t') \, \phi(t') = 0 \tag{3}$$

and we see that there is no obstacle to causality, with no further restrictions on locality.

To actually compute the CTPEA, Schwinger observed that the mean value of the field could be obtained from a generating functional

$$Z[J^+, J^-] = e^{iW[J^+, J^-]} = \langle \tilde{T}(e^{-i \int J^- \Phi})T(e^{i \int J^+ \Phi}) \rangle \tag{4}$$

where Φ is the Heisenberg field operator, the expectation value is taken with respect to the corresponding quantum state, and $T, \tilde{T}$ stand for temporal and antitemporal ordering, respectively. Indeed, if we define

$$\phi^\pm = \pm\frac{\delta W}{\delta J^\pm} \tag{5}$$

Then in the limit $J^\pm \to 0$, we obtain $\phi^+ = \phi^- = \langle\Phi\rangle$. This suggests defining Γ as the Legendre transform of W

$$\Gamma[\phi^+, \phi^-] = W[J^+, J^-] - \int (J^+\phi^+ - J^-\phi^-) \tag{6}$$

So in general we obtain

$$\frac{\delta\Gamma}{\delta\phi^\pm} = \mp J^\pm$$

and for the physical mean field

$$\left.\frac{\delta\Gamma}{\delta\phi^\pm}\right|_{\phi^+ = \phi^- = \langle\Phi\rangle} = 0 \tag{7}$$

as required. Observe that on top of Eq. (1), the quantum CTPEA obeys $\Gamma[\phi^+, \phi^-] = -\Gamma[\phi^-, \phi^+]^*$, so that the kernel $\mathcal{D}$ must be real (and so are the equations of motion) and $\mathcal{N}$ is pure imaginary ($\mathcal{N} = iN$).

The CTPEA admits a functional representation derived from the usual background field methods [22]

$$e^{i\Gamma[\phi^+,\phi^-]} = \int D\varphi^+ \, D\varphi^- \, \exp i \left[S(\varphi^+) - S(\varphi^-) \right.$$

$$\left. - \frac{\delta\Gamma}{\delta\phi^+}(\varphi^+ - \phi^+) + \frac{\delta\Gamma}{\delta\phi^-}(\varphi^- - \phi^-) \right] \quad (8)$$

The variables of integration must coincide at some very large time T_∞ in the future, and we have included the information on the quantum state in the integration measure. In our case, we have both system (φ) and environment (ψ) degrees of freedom, but we associate a mean field only to the former. The classical action S will be the sum of the system action S_s, the environment action S_e, and the interaction term S_i. In the semiclassical limit, we neglect the deviation from the mean of the system variables, and the expression for the CTPEA simplifies to

$$e^{i\Gamma[\phi^+,\phi^-]} = e^{i[S_s(\phi^+)-S_s(\phi^-)]}$$

$$\times \int D\psi^+ \, D\psi^- \, \exp i[S_e(\psi^+) + S_i(\phi^+, \psi^+)$$

$$- S_e(\psi^-) - S_i(\phi^-, \psi^-)] \quad (9)$$

where, again, the integration is nontrivial thanks to the integration measure and the future boundary conditions.

Equation (9) provides a well-defined recipe for the CTPEA, and thence finding the effective equations of motion is only a matter of computing power. The question arises of when the solutions to the semiclassical equations are relevant to the description of our experience. Mostly what we wish to do is to compute expectation values of system observables. Insofar as these observables do not involve environmental variables, their expectation values will admit representations such as

$$\mathcal{A} = \int D\phi^+ \, D\phi^- \, A[\phi^+, \phi^-]e^{i\Gamma[\phi^+,\phi^-]} \quad (10)$$

which under the saddle point approximation reduces to

$$\mathcal{A} = A[\phi, \phi] \quad (11)$$

ϕ being the solution to the mean-field equations. In other words, the semiclassical equations will be useful if our only interest is to compute expectation values for system observables, such that the integral (10) may be evaluated by saddle point methods.

The CTPEA is a powerful method to derive equations of motion for open (or effectively open) systems, which are guaranteed to be both real and

causal [23, 24]. It is possibly the most powerful method at hand to study the nonequilibrium evolution of quantum fields, especially if combined with more sophisticated resummation methods which allow us to keep track of higher Schwinger functions along with the mean field [25, 26]. But two things ought to send an alarm signal. First, if we only care about the equations of motion, we are only using a small part of the information encoded in the CTPEA; for example, the kernel $\mathcal{N}$ is irrelevant to the linearized equations (3). Second, there is no noise in the mean-field equations. We must stress that this is not only unseemly, but, insofar as the equations of motion are dissipative, it is actually *wrong*, as it violates the necessary balance between fluctuations and dissipation [27].

2.2. Where Is the Noise?

To the best of our knowledge, it was Feynman who pointed out that the untapped terms in the CTPEA contained the information about noise [19]. For simplicity, assume the CTPEA has the quadratic form (2). Then we have the identity

$$e^{i\Gamma[\phi^+,\phi^-]} = \int Dj\, \rho[j]\, \exp\left\{\frac{i}{2} \int dt\, dt'\ [\phi](t)\mathcal{D}(t,\,t')\{\phi\}(t') + i \int dt\, j(t)[\phi](t) \right\} \tag{12}$$

with

$$\langle j(t)j(t')\rangle \equiv \int Dj\, \rho[j]j(t)j(t') = N(t,\,t') \tag{13}$$

Now we can write the average (10)

$$\mathcal{A} = \int Dj\, \rho[j] \int D\phi^+\, D\phi^-\ A[\phi^+,\,\phi^-]$$

$$\times \exp\left\{\frac{i}{2} \int dt\, dt'\ [\phi](t)\mathcal{D}(t,\,t')\{\phi\}(t') + \int dt\, j(t)[\phi](t) \right\} \tag{14}$$

and use saddle point evaluation in the inner integral

$$\mathcal{A} = \int Dj\, \rho[j]\, A[\phi_j,\,\phi_j] \tag{15}$$

where ϕ_j is the solution to

$$\int dt' \, \mathcal{D}(t, t')\phi(t') = -j(t) \tag{16}$$

In this way, we have transformed the average of the observable $\mathcal{A}$ over the quantum fluctuations of the environment into the ensemble average over the realizations of the "noise" $j(t)$, at the same time upgrading the semiclassical equations to the Langevin equations (16). As expected, the relevant information on the noise (its correlation function), is given by the "useless" part of the CTPEA, namely the kernel $N = -i\mathcal{N}$.

Of course, Eq. (12) is not the only way to decompose the CTPEA into partial integrations. The point is that this particular decomposition makes physical sense. To see this, assume the interaction term in the action takes the particular form

$$S_i = \int dt \, \Xi[\psi]\phi \tag{17}$$

with $\langle \Xi \rangle = 0$ when $\phi = 0$ but otherwise arbitrary. Then [3]

$$N(t, t') = \frac{1}{2} \langle \{\Xi(t), \Xi(t')\} \rangle \tag{18}$$

the expectation value being computed at $\phi = 0$. Indeed, the Heisenberg equation for this model is

$$\frac{\delta S_s}{\delta \phi} = -\Xi[\psi] \tag{19}$$

We assume that the Heisenberg operator for the system variable is close to a c-number. Also, in the presence of a nonzero background ϕ, the operator Ξ will generally develop a nonzero expectation value $\langle \Xi \rangle_\phi$. Subtracting this, we get

$$\frac{\delta S_s}{\delta \phi} + \langle \Xi \rangle_\phi = -(\Xi - \langle \Xi \rangle_\phi) \tag{20}$$

The CTPEA, if we forget about noise, leads to the same equation with no right-hand side. As discussed above, this is not acceptable. So the question is, what is the sensible way of replacing the q-number operator in the right-hand side of Eq. (20) by a c-number stochastic source? Of course, some loss of information (especially concerning quantum coherence) is unavoidable, and in particular cases this may invalidate the whole procedure. But whenever quantum coherence is not the main concern, a Gaussian source with self-correlation as in Eq. (18) (we again neglect $\langle \Xi \rangle_\phi$, which vanishes at $\phi = 0$,

since we assume small fluctuations, but this is not essential) is the time-honored answer, and indeed the only answer compatible with the fluctuation-dissipation theorem [28].

3. NOISE-INDUCED INFLATION: WHEN NOISE MATTERS

As we have seen in the previous section, the CTPEA provides a systematic framework in which to study semiclassical evolution, taking into account at least the leading effects due to quantum fluctuations of matter fields. However, putting this framework into action is by no means a simple task. Indeed, after the observation that the CTPEA provides a simple way of deriving Einstein–Langevin equations [3], it was realized that actually carrying out the derivation is a research project in itself [4–8], to say nothing of solving those equations once derived [9]. So it is natural to wonder if noise makes such a difference as to justify this trouble.

The basic problem is that, while it would be easy to find problems where the noise level is huge, this same noisiness would lead to the suspicion that the whole semiclassical approach is breaking down. The real challenge is to find a problem where the semiclassical approximation is reliable, and noise still makes a difference. Indeed, in the case of conventional, noiseless semiclassical theory, such a problem is Hawking evaporation of large black holes: a weak effect, which puts in no jeopardy the validity of the semiclassical approximation, but whose result is utterly impossible in terms of the classical theory alone.

In this paper I will report on one such problem, a cyclic universe, provided with a cosmological constant, but prevented from inflating by the potential barrier from its own spatial curvature. In each cycle, the semiclassical effects induce a transition from one classical orbit to another; the change is small for each cycle, but overall it offers an escape route with no classical analog. Our goal is to compute the average escape probability due to semiclassical effects.

This problem has an important precedent, the calculation of the tunneling amplitude due to Vilenkin [29]. It is important to realize the similarities and differences between these two approaches. Vilenkin's calculation was fully quantum gravitational, but it contemplated only the effects of the gravitational field. It was tacitly assumed that, if any matter fields were present, they would at most affect the prefactor of the exponentially suppressed tunneling probability [30–35]. Our calculation is only semiclassical, but we put the stress precisely on the effects of the matter fields. From the point of view of the usual instanton approach, we could say ours is a highly nonperturbative evaluation of the prefactor, since we go well beyond the test field one-loop approximation. The result is that the escape probability due to the fluctuations

in matter fields is at least as large as the tunneling one, suggesting that in nature both must be taken into account. We shall not discuss subsequent developments related to Vilenkin's proposal [36].

Since the calculation of the escape probability due to semiclassical effects is discussed in some detail elsewhere [1], here I shall only give a general discussion of the several steps involved, the peculiar difficulties of each one, and how they could be overcome.

3.1. The Model

Our model is based on a spatially closed, homogeneous Friedmann–Robertson–Walker (FRW) model, with a metric

$$ds^2 = a^2(t)(-dt^2 + \tilde{g}_{ij}(x^k)\, dx^i\, dx^j), \qquad i, j, k = 1, \ldots, n-1 \quad (21)$$

where $a(t)$ is the cosmological scale factor, t is the conformal time, and $\tilde{g}_{ij}(x^k)$ is the metric of an $(n-1)$-sphere of unit radius. Since we will use dimensional regularization we work, for the time being, in n dimensions. Matter is described by a quantum scalar field $\Phi(x^\mu)$, where the Greek indices run from 0 to $n-1$. The classical action for this scalar field in the spacetime background described by the above metric is

$$S_m = -\int dx^n \,\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\Phi^*\partial_\nu\Phi + \left(\frac{n-2}{4(n-1)} + \nu\right)R\Phi^*\Phi \right] (22)$$

where $g_{00} = a^2$, $g_{0i} = 0$, $g_{ij} = a^2\tilde{g}_{ij}$, g is the metric determinant, ν is a dimensionless parameter coupling the field to the spacetime curvature ($\nu = 0$ corresponds to conformal coupling), and R is the curvature scalar which is given by

$$R = 2(n-1)\frac{\ddot{a}}{a^3} + (n-1)(n-4)\frac{\dot{a}^2}{a^4} + (n-1)(n-2)\frac{1}{a^2} \quad (23)$$

where an overdot means derivative with respect to conformal time t. Let us now introduce a conformally related field Ψ

$$\Psi = \Phi a^{(n-2)/2} \quad (24)$$

the time-dependent function $U(t)$

$$U(t) = -\nu a^2(t)R(t) \quad (25)$$

and the d'Alambertian $\Box = -\partial_t^2 + \Delta^{(n-1)}$ of the static metric $\tilde{ds}^2 = a^{-2}\,ds^2$. The action may be written as

$$S_m = \int dt\, dx^1 \ldots dx^{n-1} \ \sqrt{g} \left[\Psi * \Box \Psi - \frac{(n-2)^2}{4} \Psi * \Psi + U(t)\Psi * \Psi \right] \qquad (26)$$

When $\nu = 0$ this is the action of a scalar field Ψ in a background of constant curvature. The quantization of this field in that background is trivial in the sense that a unique natural vacuum may be introduced, the "in" and "out" vacua coincide, and there is no particle creation [2]. This vacuum is, of course, conformally related to the physical vacuum; see (24). The time-dependent function $U(t)$ will be considered as an interaction term and will be treated perturbatively. Thus we will make perturbation theory with the parameter ν, which we will assume small.

To carry on the quantization we will proceed by mode separation, expanding $\Psi(x^\mu)$ in terms of the $(n-1)$-dimensional spherical harmonics $Y_k^l(x^i)$. The coefficients $\Psi_k^l(t)$ are just functions of t (one-dimensional fields), and for each set (l, k) we may introduce two real functions $\phi_k^l(t)$ and $\tilde{\phi}_k^l(t)$ defined by

$$\Psi_k^l \equiv \frac{1}{\sqrt{2}} (\phi_k^l + i\tilde{\phi}_k^l) \qquad (27)$$

The action becomes the sum of the actions of two independent sets formed by an infinite collection of decoupled time-dependent harmonic oscillators

$$S_m = \frac{1}{2} \int dt \sum_{k=1}^{\infty} \sum_k [(\dot{\phi}_k^l)^2 - M_k^2(\phi_k^l)^2 + U(t)(\phi_k^l)^2] + \ldots \qquad (28)$$

where the dots stand for an identical action for the real one-dimensional fields $\tilde{\phi}_k^l(t)$.

We will consider, from now on, the action for the one-dimensional fields ϕ_k^l only. The field equations, for the one-dimensional fields $\phi_k^l(t)$ are, from (28),

$$\ddot{\phi}_k^l + M_k^2\phi_k^l = U(t)\phi_k^l \qquad (29)$$

which in accordance with our previous remarks will be solved perturbatively on $U(t)$. The solutions of the unperturbed equation can be written as linear combinations of the normalized positive- and negative-frequency modes f_k and f_k^*, respectively, where

$$f_k(t) = \frac{1}{\sqrt{2M_k}} \exp(-iM_k t) \qquad (30)$$

3.2. Closed-Time-Path Effective Action

We are now in the position to compute the regularized semiclassical CTP effective action. This involves a careful consideration of the infinities arising in perturbation theory, but after the dust settles, the result is

$$\Gamma_{\text{CTP}}[a^\pm] = S^R_{g,m}[a^+] - S^R_{g,m}[a^-] + S^R_{IF}[a^\pm] \tag{31}$$

where the regularized gravitational and classical matter actions are

$$S^R_{g,m}[a] = \frac{2\pi^2}{l^2_P} \int dt\, 6a^2 \left(\frac{\ddot{a}}{a} + 1\right) - 2\pi^2 \int dt\, a^4 \Lambda^* + \frac{1}{16} \int dt\, U^2_1(t) \ln(a\mu_c) \tag{32}$$

and the influence action is

$$S^R_{IF}[a^\pm] = \frac{1}{2} \int dt\, dt'\, \Delta U(t) H(t - t')\{U(t')\}$$

$$+ \frac{i}{2} \int dt\, dt'\, \Delta U(t) N(t - t')\, \Delta U(t') \tag{33}$$

where we have defined

$$\Delta U = U^+ - U^-, \qquad \{U\} = U^+ + U^- \tag{34}$$

Computing the kernels H and N involves the consideration of Feynman graphs, where the internal legs represent propagators for a particle in a closed space. This calculation may be carried out exactly, but the result is that, unless for orbits with very small amplitude, the effect of spatial curvature is not really important. It is convenient to compute these kernels as in a spatially flat FRW universe with the same radius, which amounts to considering a continuous, rather than a discrete, spectrum of modes. The result is

$$N(u) = \int_0^\infty dk \cos 2ku = \frac{\pi}{16}\, \delta(u) \tag{35}$$

$$H(u) = \frac{1}{8} \text{Pf}\left[\frac{\theta(u)}{u}\right] + \frac{\gamma + \ln \mu_c}{8}\, \delta(u) \tag{36}$$

The distribution $\text{Pf}(\theta(u)/u)$ should be understood as follows. Let $f(u)$ be an arbitrary tempered function; then

$$\int_{-\infty}^\infty du\, \text{Pf}\left[\frac{\theta(u)}{u}\right] f(u) = \lim_{\epsilon \to 0^+} \left(\int_\epsilon^\infty du\, \frac{f(u)}{u} + f(0) \ln \epsilon\right) \tag{37}$$

The approximation of substituting the exact kernels by their flat-space counterparts is clearly justified when the radius of the universe is large, which is when the semiclassical approximation works best.

The imaginary part of the influence action is known [3, 5, 6, 37–40] to give the effect of a stochastic force on the system, and we can introduce an improved semiclassical effective action,

$$S_{\text{eff}}[a^{\pm}; \xi] = S^R_{g,m}[a^+] - S^R_{g,m}[a^-] + \frac{1}{2} \int dt\, dt'\; \Delta U(t) H(t - t') \{U(t')\}$$

$$+ \int dt\, \xi(t)\, \Delta U(t) \tag{38}$$

where $\xi(t)$ is a Gaussian stochastic field defined by the following statistical averages:

$$\langle \xi(t) \rangle = 0, \qquad \langle \xi(t)\xi(t') \rangle = N(t - t') \tag{39}$$

The kernel H in the effective action gives a nonlocal effect (due to particle creation), whereas the source ξ gives the reaction of the environment on the system in terms of a stochastic force.

3.3. The Einstein–Langevin Equation

The dynamical equation for the scale factor $a(t)$ can now be found from the effective action (38) in the usual way, that is, by functional derivation with respect to $a^+(t)$ and then equating $a^+ = a^- \equiv a$. These equations include the backreaction of the quantum field on the scale factor; they improve the semiclassical equation by taking into account the fluctuations of the stress-energy tensor of the quantum field [15, 16]. However, they also lead to the typical nonphysical runaway solutions due to the higher order time derivatives involved in the quantum correction terms.

To avoid such spurious solutions we use the method of order reduction [41]. In this method one assumes that the equations obtained from the CTPEA are perturbative, the perturbations being the quantum corrections. To leading order the equation reduces to the classical equation, which, in terms of scaled variables

$$b(t) \equiv \frac{\sqrt{24\pi}}{l_P}\, a(t), \qquad \Lambda \equiv \frac{l_P^4}{12\pi^2}\, \Lambda^* \tag{40}$$

reads

$$\ddot{b} + b\left(1 - \frac{1}{6}\Lambda b^2\right) = O(v) \tag{41}$$

The terms with $\ddot{b}$ or with higher time derivatives in the quantum corrections are then substituted using recurrently the classical equation (41). In this form the solutions to the semiclassical equations are also perturbations of the classical solutions. Thus, by functional derivation of (38), we can write the stochastic semiclassical backreaction equation as

$$\dot{p} = -V'(b) - \delta V'(b) + F(b, p, t) + J(\xi, b, p) \tag{42}$$

where a prime means a derivative with respect to b, and we have introduced $p \equiv \dot{b}$. The classical potential $V(b)$ is

$$V(b) = \frac{1}{2} b^2 - \frac{\Lambda}{24} b^4 \tag{43}$$

An schematic plot of this potential is given in Fig. 1.

The remaining terms in Eq. (42) represent the quantum corrections. The first one is purely local

$$\delta V(b) = -\frac{3v^2\Lambda}{4} \left[\frac{1}{2} b^2 - \frac{\Lambda}{48} b^4 - p^2 \ln(b\bar{\mu}) \right] \tag{44}$$

where we have already implemented order reduction.

From this point on, we shall disregard the local quantum correction to the potential, $\delta V(b)$. In the region where semiclassical theory is reliable, this is only a very small correction to the classical potential; moreover, we are concerned with such phenomena where the semiclassical behavior is qualitatively different from the classical one, which is not the case for these corrections.

The term $F(b, p, t)$ involves nonlocal contributions and may be written as

$$F(b, p, t) = -\frac{\partial U}{\partial b} I - \frac{d^2}{dt^2} \left(\frac{\partial U}{\partial \dot{b}} I \right) = 6v \left\{ \frac{d^2}{dt^2} \frac{1}{b} - \frac{\dot{b}}{b^2} \right\} I \tag{45}$$

where $I(b, p, t)$ is defined by

$$I(b, p, t) \equiv \int_{-\infty}^{\infty} dt' \, H(t - t')U(t') \tag{46}$$

and

$$U(t) = -6v \left(\frac{\dot{b}}{b} + 1 \right) \tag{47}$$

After order reduction, $U(t')$ must be evaluated on the classical orbit with Cauchy data $b(t) = b$, $p(t) = p$, whereby it reduces to $U = -\Lambda v b^2$. Observe that, in fact, this approximation makes the equation of motion local in time, though no longer Hamiltonian. Finally, the function J is the noise given by

$$J(\xi, b) = 6v \left\{ \frac{d^2}{dt^2} \left(\frac{\xi}{b} \right) - \frac{\dot{b}\xi}{b^2} \right\}$$

and, after order reduction, by

$$J(\xi, b, p) = 6\nu \left[\frac{\xi}{b} - \frac{2\dot{\xi}p}{b^2} + \frac{2\xi V'(b)}{b^2} + \frac{2\xi p^2}{b^3} \right] \tag{48}$$

with $\xi(t)$ defined in (39) in terms of the noise kernel.

3.4. The Classical Orbits

Before continuing, it is convenient to pause and consider the classical orbits, as described by Eq. (41). They represent a particle moving in the one-dimensional potential well Eq. (43), plotted schematically in Fig. 1. This evolution preserves the Wheeler–DeWitt operator (energy, for short)

$$H(b, p) = \frac{1}{2} p^2 + V(b) \tag{49}$$

Physically, the value of H on a given orbit is the energy density of radiation present besides the cosmological constant.
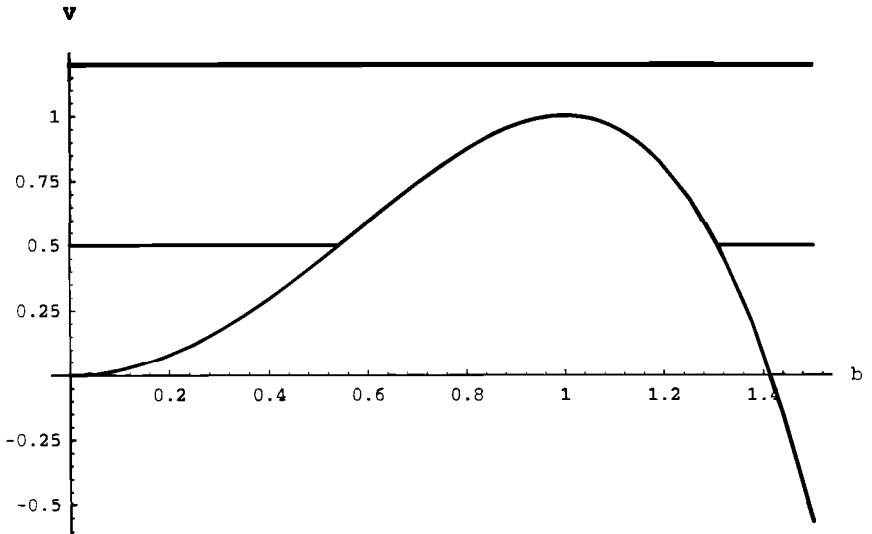
Fig. 1. A schematic plot of the classical potential; it vanishes as b^2 when $b \to 0$, it has a maximum, and decreases without bound for larger universes. Classical evolution preserves the Wheeler–DeWitt operator $H = p^2/2 + V(b)$. If H exceeds the maximum of the potential, the corresponding orbit either expands forever or collapses to the singularity. For lower H, the classical orbit bounces off the outer classical turning point. For positive H below the maximum of the potential, we have periodic bounded orbits representing an eternal cyclical universe.

Figure 2 is a schematic representation of the classical phase space. There is a stable fixed point (*nothing*) at $b = p = 0$. This is the starting point of Vilenkin's calculation. There is an unstable fixed point corresponding to

$$H = E_s = \frac{3}{2\Lambda}; \qquad b = 2\sqrt{E_s}; \qquad p = 0 \qquad (50)$$

This is an Einstein-type static universe. When $H > E_s$, orbits are free to expand forever.

For $H < E_s$, we have two types of orbits. Those outside the well contract at first, until they reach the classical turning point b_+ and bounce off. The de Sitter universe, with $H = 0$, belongs to this family. In their final stages, these orbits are essentiallly identical to the ever-expanding ones (cosmic no-hair theorem). Orbits inside the well with $0 < H < E_s$ bounce eternally between the turning points $\pm b_-$ (there is no problem with a negative radius of the universe, since only b^2 has a physical meaning; we may also think of $b = 0$ as a perfectly reflecting boundary). The actual location of the turning points is
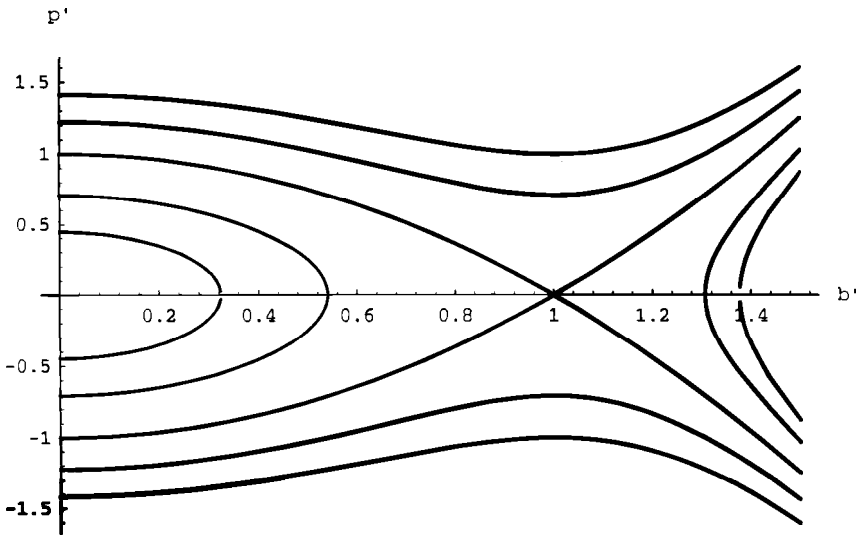
Fig. 2. A sketch of the classical phase space. Only half is shown, the other half being the mirror image. We can see the stable (elliptic) fixed point at the origin, and one of the unstable (hyperbolic) fixed points. The separatrices connect the unstable points to each other, and divide the region of periodic motion (within) from the region of unbound motion (outside). The normalization is $b' = b/2\sqrt{E_s}$, $p' = p/\sqrt{2E_s}$.

$$b_\pm^2 = 4E_s \left[1 \pm \sqrt{1 - \frac{E}{E_s}} \right] \qquad (51)$$

The frequency Ω of oscillation is 1 for $H \ll E_s$, and vanishes as $H \to E_s$. This limiting value corresponds to two orbits, exponentially departing from and approaching to the unstable fixed points, the so-called separatrices.

To fix ideas, let us adopt for Λ a value consistent with grand unified scale inflation, which in natural units means $\Lambda \sim 10^{-12}$. Then the value of the separatrix energy is very high, $E_s \sim 10^{12}$. Our problem is to find a way for a typical universe ($H \sim 1$), trapped within the well to climb out of it and inflate. As we shall see, this is possible thanks to the combination of the diffusive effect of quantum fluctuations and a runaway instability associated with particle creation.

It ought to be clear that the final state of this evolution will be very different than in the instanton approach. In the quantum calculation, the universe emerges from under the barrier as an empty, $H = 0$, de Sitter universe. In our calculation, the universe goes above the barrier and emerges with a large amount of radiation $H = E_s$, corresponding to particles created while inside the well. Physically, however, the difference is minor, as this energy gets diluted in a few e-foldings by the inflationary expansion.

3.5. From Langevin to Kramers

Now we want to determine the probability that a universe starting at the potential well goes over the potential barrier into the inflationary stage. The magic of the CTPEA has turned an originally quantum problem into a statistical mechanical one, indeed a classic problem associated with the name of Kramers [42]. Observe that we are not interested in the features of solutions associated with peculiar realizations of the noise, but rather in a noise-averaged observable. Therefore, it is convenient to perform the noise average at the outset, introducing the distribution function

$$f(b, p, t) = \langle \delta(b(t) - b)\delta(p(t) - p) \rangle \qquad (52)$$

where $b(t)$ and $p(t)$ are solutions of Eq. (42) for a given realization of $\xi(t)$, b and p are points in the phase space, and the average is taken both with respect to the initial conditions and to the history of the noise. After some standard manipulations we arrive at the so-called Kramers' equation [43]

$$\frac{\partial f}{\partial t} = \{H, f\} - \frac{\partial}{\partial p}[F(b, p, t)f] - \frac{\partial}{\partial p}\Phi \qquad (53)$$

where the curly brackets are Poisson brackets, i.e.,

$$\{H, f\} = -p(\partial f/\partial b) + V'(b)(\partial f/\partial p)$$

and

$$\Phi = -\frac{\pi v^2 \Lambda^2}{4} b^2 \frac{\partial f}{\partial p} \qquad (54)$$

This term will be called the diffusion term since it depends on the stochastic field $\xi(t)$.

We notice that in the absense of a cosmological constant, we get no diffusion. This makes sense, because in that case the classical trajectories describe a radiation-filled universe. Such universe would have no scalar curvature, and so it should be insensitive to the value of v as well.

3.6. From Kramers to Fokker–Planck

In the usual statement of Kramers' problem, the system is described by a single variable x and obeys a Fokker–Planck equation [44]

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial x} \left[f \frac{\partial F}{\partial x} + T \frac{\partial f}{\partial x} \right] \qquad (55)$$

where T is the temperature (which fixes the sign of the diffussion term) and F is the free energy (rather than a potential). Activation is studied from the properties of the steady solutions of this equation, and the answer is the so-called Arrhenius formula

$$P \sim e^{-F_{\max}/T} \qquad (56)$$

where $F_{\max}$ is the value at the peak of the free energy barrier.

Our Kramers equation is certainly more involved than Eq. (55), because it describes other phenomena besides tunneling. Basically, there are three things going on. Given a generic distribution function f, its dynamics consists mostly of the representative phase-space points being dragged along the classical orbits with a time scale of the order of a typical period. On a larger time scale, we have the diffusion process, which makes f evolve toward a quasiequilibrium, steady solution. Finally, there is activation, on an even larger time scale.

Since our concern is this third process only, it is convenient to get rid of the two faster ones. We get rid of classical transport by defining a new distribution function which counts the number of universes on a given classical orbit, rather than on a phase-space cell. This new distribution function does not tell us where in the orbit we are, but we do not need that to study activation. We achieve this by transforming the problem to action–angle variables and averaging over the angles [45]. Finally, we get rid of the

approach to quasiequilibrium by assuming a steady solution from the beginning.

The averaged Kramers equation becomes

$$\frac{\partial f}{\partial t} = \frac{\pi v^2 \Lambda^2}{4} \frac{\partial}{\partial J} \left\{ \frac{\mathbf{D}(J)}{\Omega} \frac{\partial f}{\partial J} - \mathbf{S}f \right\} \tag{57}$$

where

$$\mathbf{D}(J) = \frac{1}{2\pi\Omega} \int_0^{2\pi} d\theta \, b^2 p^2 \tag{58}$$

$$\mathbf{S}(J) = \frac{-1}{4\pi^2} \int_0^{2\pi/\Omega} dt \left(\frac{d}{dt} b^2(t) \right) \text{pf} \int_0^{\infty} \frac{du}{u} b^2(t-u) \tag{59}$$

This equation may be written as a continuity equation $\partial_t f + \partial_J K = 0$, where the probability flux K may be identified directly from (57). We see that, as in Kramers' problem, stationary solutions with positive flux K_0 should satisfy

$$\frac{\mathbf{D}(J)}{\Omega} \frac{\partial f}{\partial J} - \mathbf{S}(J)f = -\frac{4}{\pi v^2 \Lambda^2} K_0 \tag{60}$$

From now on it is more convenient to use the energy E as a variable instead of J, where $E = H(J)$. Both $\mathbf{D}$ and $\mathbf{S}$ individually behave as E^2 times a smooth function of E/E_s, and their ratio is relatively slowly varying. At low energy, we find $\mathbf{D} \sim E^2/2$ and $\mathbf{S} \sim E^2/4$. As we approach the separatrix, $\mathbf{D} \to 0.96 \, E_s^2$ and $\mathbf{S} \to 1.18 \, E_s^2$. Meanwhile, the ratio of the two goes from 0.5 to 1.23. This means that we can write the equation for stationary distributions as

$$\frac{\partial f}{\partial E} - \beta \, (E)f = -\frac{4}{\pi v^2 \Lambda^2 g(E)} \left(\frac{K_0}{E^2} \right) \tag{61}$$

where β and g are smooth order-one functions. There is a fundamental difference with respect to Kramers' problem, namely the sign of the second term in the left-hand side. In the cosmological problem, the effect of nonlocality is to favor diffusion rather than hindering it. We may understand this as arising from a feedback effect associated with particle creation [11].

3.7. The Activation Amplitude

Figure (3) is an out-of-scale, schematic plot of the solution of Eq. (61). For $E \ll 1$, the solution diverges as $1/E$; for $E \geq 1$, and for 12 decades thereafter, it grows exponentially. Of course, our analysis doeas not hold beyond the separatrix, but it can be shown that f turns around there, decaying as a power of E as $E \to \infty$.
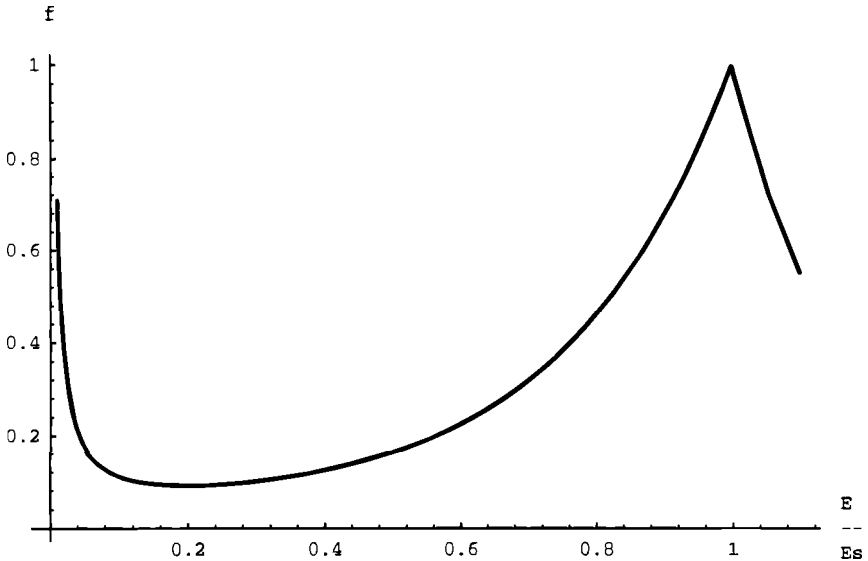
Fig. 3. A qualitative plot of the equilibrium distribution function as a function of E/E_s. One can appreciate the divergence toward the origin, the exponential rise toward the separatrix, and the falling off in the inflationary region. For generic values of the cutoff, the area under the curve is dominated by the peak at the separatrix.

This behavior cannot be extrapolated all the way to zero as it would make f nonintegrable. However, we must notice that neither our treatment (i.e., the neglect of logarithmic potential corrections) nor semiclassical theory generally is supposed to be valid arbitrarily close to the singularity. Thus we shall assume that the pathological behavior of Eq. (61) near the origin will be absent in a more complete theory, and apply it only from some lowest energy $E_\delta \sim 1$ on. There are still 12 orders of magnitude between E_δ and E_s.

We may now estimate the flux by requesting that the total area below the distribution function should not exceed unity. Unless the lower cutoff E_δ is very small (it ought to be exponentially small on E_s to invalidate our argument) the integral is dominated by the peak around E_s, and we obtain

$$K_0 \le (\text{prefactor}) \cdot \exp[-\beta(E_s)E_s] \qquad (62)$$

The prefactor depends on Λ, ν, $g(1)$, $\beta(1)$, σ, and the details of the peak shape, but only logarithmically on the cutoff. So we can take this as a bona fide prediction of noisy semiclassical theory. Using $E_s = 3/(2\Lambda)$, $\beta(E_s) = 1.23$, we get

$$K_0 \le (\text{prefactor}) \cdot \exp(-1.84/\Lambda) \qquad (63)$$

This is the semiclassical result to be compared against the instanton calculations [29], which yield

$$P \sim \exp(-8/\Lambda) \tag{64}$$

We see that the probability of noise activation is indeed larger than that of quantum tunneling.

A final pertinent question is, was noise truly necessary? After all, one could imagine there would be particle creation in each cycle, and the accumulation of these particles alone would make inflation progressively easier to achieve. Of course, in the absense of diffusion our argument would need to be entirely redrawn; for example, it is not longer clear than an arbitrary f would tend toward a steady solution, or on which time scales, nor that the steady solution would have an acceptable behavior beyond the separatrix. However, one could try to see what happens if one simply kills the diffusion term in Eq. (61). The equation still admits a solution, but the divergence at the origin gets worse ($1/E^2$), and indeed the flux seems to depend solely on the value of the infrared cutoff, $K_0 \sim E_\delta$, which cannot be predicted within the semiclassical theory. It is only the combination of particle creation and diffusion which sets up the mechanism by which we get a definite result.

On the other hand, an infinity of cycles is not truly necessary. There is a similar result in models where the universe is restricted to a single cycle; the activation probability is somewhat lower, but still higher than the tunneling one. We can understand this by analogy to the problem of black hole formation in a box, where a big hole can form by slow accretion of smaller holes or by a sudden, single large fluctuation. The difference is, of course, that in the black hole case the latter is more likely than the former [46].

4. FINAL REMARKS

We have reported on a cosmological process where quantum-induced noise and particle creation combine to yield a behavior notoriously different than expected from classical theory, or even conventional, deterministic semi-classical gravity. The strength of the effect is indeed comparable to a purely quantum calculation, which shows that treating matter as test fields in quantum gravity may not be justified. We believe this work is meaningful on at least three different levels:

(a) Of course, our results are most important as a step forward in the development of stochastic, semiclassical cosmological models. By now, the mathematical and physical basis of such models is rather well understood, but the development of actual models and the gathering of concrete predictions is lagging behind. Our calculation has demanded the application of a number of techniques which are not common tools of the trade in cosmology, and could serve as an example for future projects.

(b) The relevance to cosmology may seem minor, since it does not seem likely that our universe is spatially closed. However, the situation changes

if the original question is rephrased as: Is it possible that a horizon-size, overdense region in the early universe, with a homogeneous but sub-Planckian value of the inflaton field, may avoid collapse and inflate? Clasically, the answer is no, and this negative result may well be the bane of inflationary models [47]. Our results suggest that semiclassically things may turn around.

(c) Finally, it has been observed that noise and dissipation are generic to all effective theories [40]. So we must expect that similar results will be found in the analysis of nucleation phenomena in other effective theories as well, especially in quantum field theories [48], and in out-of-equilibrium situations. Indeed, an approach such as ours seems to be the only way of analyzing tunneling in situations where the environment changes on time scales comparable to the time it takes to nucleate a bubble, an essentially virgin field right now.

We continue our research on all these levels, and hope to report soon on new results.

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